ARPM Certificate

Level 1

Example

This is an example of examination sheet for the Certificate in Advanced Risk and Portfolio Management (ARPM Certificate) - Level 1.

The exercises below test the following modules from the body of knowledge:
- Financial engineering for investment
- Data science for finance.

The exam is graded according to the breakdown policy specified on the examination sheet. The grade for the exam is a number L_1 in the interval $[0, 1]$, computed via an increasing function $L_1(p)$, where p is the total number of points that you collect by solving the exercises below. The minimum score to pass is 0.65.

Exercise 1 True or false? Justify your answer *[Tot. 30 points]*

i) *[3 points]* The security market line equation holds only under the assumption of a normal market.

ii) *[3 points]* In a linear pricing framework, the payoff of a financial instrument at a future horizon is regarded as a random variable, and the current value is given by the corresponding expectation.

iii) *[3 points]* A reasonable guess for the price of an equity share tomorrow is the sample average over a time series of past prices.

iv) *[3 points]* The yield to maturity $Y_t(\tau)$ at a given time t is the (rescaled) compounded return of a zero coupon bond from time t to the maturity date.

v) *[3 points]* The 10-year yield to maturity of the zero swap curve, sampled over daily time steps, is approximately independent and identically distributed through time.

vi) *[3 points]* If the 10-year yield to maturity follows a stationary Ornstein-Uhlenbeck process, the square-root rule does not apply, i.e. the propagation of the standard deviation is not proportional to the square-root of the time to the horizon.

vii) *[3 points]* If the yield curve follows a multivariate Ornstein-Uhlenbeck (MVOU) process, the conditional distribution of the yields to maturity at any future horizon is multivariate normal.

viii) *[3 points]* It is possible to build a principal-component linear factor model on $\bar{n} \equiv 6$ target variables using $\bar{k} = 9$ factors.

ix) *[3 points]* If the supervised probabilistic prediction for a univariate output X and input Z is a conditional Bernoulli distribution, $X|z \sim Bernoulli(p(z))$, then the conditional probability of the positive outcome $p(z)$ is a supervised point prediction.

x) *[3 points]* Regression linear factor models are unsupervised autoencoders.

Exercise 2 Scenario-probability distributions, historical estimation, location-dispersion ellipsoid *[Tot. 15 points]*

Consider a bivariate time series $\{\boldsymbol{\epsilon}_t\}_{t=1}^{\bar{t}}$ with $\bar{t} = 3$ observations

$$\{\underset{\substack{\uparrow\\ \boldsymbol{\epsilon}_1}}{\begin{pmatrix} -1 \\ 1 \end{pmatrix}}, \quad \underset{\substack{\uparrow\\ \boldsymbol{\epsilon}_2}}{\begin{pmatrix} 0 \\ -1 \end{pmatrix}}, \quad \underset{\substack{\uparrow\\ \boldsymbol{\epsilon}_3}}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}\}. \tag{1}$$

i) *[3 points]* Compute the historical expectation $\hat{\boldsymbol{m}}_{\boldsymbol{\varepsilon}}^{Hist}$ and covariance $\hat{\boldsymbol{s}}_{\boldsymbol{\varepsilon}}^{2Hist}$.

ii) *[6 points]* Draw the scatter plot of (1) and sketch the location-dispersion ellipsoid $\mathfrak{E}(\hat{\boldsymbol{m}}_{\boldsymbol{\varepsilon}}^{Hist}, \hat{\boldsymbol{s}}_{\boldsymbol{\varepsilon}}^{2Hist})$ (center and principal axes).

iii) *[3 points]* Compute the historical with flexible probabilities expectation $\hat{\boldsymbol{m}}_{\boldsymbol{\varepsilon}}^{HFP}$ and covariance $\hat{\boldsymbol{s}}_{\boldsymbol{\varepsilon}}^{2HFP}$, given the probabilities

$$p_1 = 20\%, \quad p_2 = 20\%, \quad p_3 = 60\%.$$

iv) *[3 points]* Compute the scenario-probability distribution of the transformed random variable $\boldsymbol{\varepsilon}^2 - \sin(\frac{\pi}{2}\boldsymbol{\varepsilon})$.

Exercise 3 The Checklist applied to stocks: steps 1 to 5 with copula-marginal approach *[Tot. 30 points]*

Consider as risk drivers the log-values of the 500 stocks in the S&P 500.
Assume that each risk driver follows a GARCH(1,1) process over a daily time step.
Assume that the GARCH(1,1) shocks are independent and identically distributed through time. Model their marginal distributions via the respective historical distributions and the (static) copula as a t variable with $\nu = 5$ degrees of freedom.
Summarize in detail, using pseudo-code, the steps that you would apply to find the joint distribution of the P&L's of the 500 stocks from today to tomorrow.

Exercise 4 Non-parametric cross-sectional linear factor models *[Tot. 25 points]*

Consider a cross-sectional linear factor model for a ($\bar{n} = 2$)-dimensional target variable $\boldsymbol{X}$ with $\bar{k} = 1$ factor Z^{CS}

$$\boldsymbol{X} = \boldsymbol{\alpha} + \boldsymbol{\beta} Z^{CS} + \boldsymbol{U}. \tag{2}$$

Recall that the cross-sectional linear factor model (2) solves the following problem

$$(\boldsymbol{\alpha}, \boldsymbol{\beta}, Z^{CS}) \equiv \underset{(\boldsymbol{a},\boldsymbol{b},F)\in\mathcal{C}}{\operatorname{argmax}} \mathcal{R}^2_{\boldsymbol{\sigma}^2}\{\boldsymbol{a} + \boldsymbol{b}F || \boldsymbol{X}\} \tag{3}$$

where the objective is the multivariate r-squared

$$\mathcal{R}^2_{\boldsymbol{\sigma}^2}\{\boldsymbol{Y}||\boldsymbol{X}\} = 1 - \frac{\mathbb{E}\{||\boldsymbol{\sigma}^{-1}(\boldsymbol{Y} - \boldsymbol{X})||^2\}}{\operatorname{tr}(\mathbb{C}v\{\boldsymbol{\sigma}^{-1}\boldsymbol{X}\})} \tag{4}$$

and the constraints read

$$\mathcal{C} : \begin{cases} \boldsymbol{b} = \boldsymbol{\beta} \\ F = \boldsymbol{c}\boldsymbol{X} \\ \boldsymbol{a} = (\mathbb{I}_{\bar{n}} - \boldsymbol{\beta}\boldsymbol{c})\mathbb{E}\{\boldsymbol{X}\} \end{cases}. \tag{5}$$

Assume that the target variables have the following scenario-probability distribution

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim \{\left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right), 25\%, \quad \left(\begin{smallmatrix} 2 \\ 1 \end{smallmatrix}\right), 25\%, \quad \left(\begin{smallmatrix} 0 \\ -1 \end{smallmatrix}\right), 25\%, \quad \left(\begin{smallmatrix} -1 \\ 0 \end{smallmatrix}\right), 25\%\},$$

that the observable loadings read

$$\boldsymbol{\beta} \equiv (1, 0)',$$

and the scale matrix specifying the r-squared (4) reads

$$\boldsymbol{\sigma}^2 \equiv \left(\begin{smallmatrix} 1.25 & 0.5 \\ 0.5 & 0.5 \end{smallmatrix}\right). \tag{6}$$

i) *[3 points]* Compute the shift parameter $\boldsymbol{\alpha} \equiv (\alpha_1, \alpha_2)'$ and the factor-construction parameter $\boldsymbol{\gamma} \equiv (\gamma_1, \gamma_2)$, which correspond to the optimal $\boldsymbol{a}$ and $\boldsymbol{c}$ in problem (3)-(5), respectively.

ii) *[5 points]* Compute the joint distribution of residuals and factor $(U_1, U_2, Z^{CS})'$.

iii) *[2 points]* Show that this cross-sectional linear factor model is systematic, but *not* idiosyncratc.

iv) *[5 points]* Compute the optimal r-squared $\mathcal{R}^2_{\boldsymbol{\sigma}^2}\{\boldsymbol{\alpha} + \boldsymbol{\beta}Z^{CS}||\boldsymbol{X}\}$.

v) *[5 points]* Suppose now arbitrary loadings $\boldsymbol{\beta} \equiv (\beta_1, \beta_2)'$ and scale matrix as in (6). Explain why this cross-sectional linear factor model is systematic, regardless of the choice of $\boldsymbol{\beta}$.

vi) *[5 points]* Suppose arbitrary loadings $\boldsymbol{\beta} \equiv (\beta_1, \beta_2)'$ and scale matrix as in (6). Explain why the optimal r-squared reads $\mathcal{R}^2_{\boldsymbol{\sigma}^2}\{\boldsymbol{\alpha} + \boldsymbol{\beta}Z^{CS}||\boldsymbol{X}\} = 0.5$, regardless of the choice of $\boldsymbol{\beta}$.