ARPM Certificate

Level 2

Example with solutions

Solution 1 True or false? Justify your answer

i) *[3 points]* A random variable X strongly dominates another random variable Y if and only if X dominates Y weakly as well.

False. Strong dominance implies weak dominance, but the converse does not hold.

ii) *[3 points]* If an index of satisfaction (or a risk measure) is consistent with weak dominance, then it is also consistent with strong dominance.

True. Consistency with weak dominance is a stronger requirement than consistency with strong dominance. See here.

iii) *[3 points]* The ex-ante performance $Y_{\boldsymbol{h}}$, say the P&L or the linear return or the excess linear return of a portfolio, is always affine in terms of the holdings $\boldsymbol{h}$.

True. The P&L, the linear return and the excess return are all linear combinations of the instruments ex-ante P&L in terms of the standardized holdings $\tilde{\boldsymbol{h}}$, which are in turn an affine transformation of the holdings $\boldsymbol{h}$. See here.

iv) *[3 points]* Quantile-based indices of satisfaction or risk measures for a buy-and-hold portfolio, such as the VaR and the CVaR, are all positive homogeneous of degree 1 in the standardized holdings.

True. Using the quantile of a positive affine transformation, it's easy to show that quantile-base indices are positive homogeneous of degree 1 in the standardized holdings. See here.

v) *[3 points]* The mean-variance trade-off is an index of satisfaction which is popular and commonly used in practice because it satisfies many preferrable properties such as monotonicity, positive homogeneity and co-monotonic additivity.

False. The mean-variance trade-off does not satisfy any of the mentioned properties. The mean-variance trade-off is popular only because it is easy to calculate and interpret.

vi) *[3 points]* The buy-and-hold strategy stemming from the expected utility maximization has a concave payoff profile.

False. The buy-and-hold strategy has a linear payoff profile. See here.

vii) *[3 points]* In the Black-Litterman approach, the views are statements on the covariances of the returns on some portfolios.

False. In the Black-Litterman approach the views are linear statements on the expected returns. See here.

viii) *[3 points]* The signal characteristic portfolio is a pseudo-inverse of the signal beta.

True. See here.

ix) *[3 points]* In the backward step-wise selection routine we build the optimal combination by removing step by step the elements which give rise to the lowest value of the objective function.

False. In the backward step-wise selection routine we build the optimal combination by removing step by step the integers n whose complementary combination $\boldsymbol{c}_k^{*backward} \setminus \{n\}$ has the highest objective . See here.

x) *[3 points]* When scheduling to optimally liquidate a parent order, it is most natural to plan the schedule in tick time since historical trades and quotes are recorded in tick time.

False. Volume time is used in order scheduling.

Solution 2 Construction: embedding views and advanced portfolio construction *[Tot. 25 points]*

Consider as daily risk drivers the log-values of the 500 stocks in the S&P 500.
Assume that the risk drivers follow a multivariate random walk process with multivariate normal shocks over a daily time step.
Assume that we have the following partial views on the invariants distribution

$$f_{\boldsymbol{\varepsilon}} \in \mathcal{V}_{\boldsymbol{\varepsilon}} : \quad \begin{cases} \mathbb{E}^{f_{\boldsymbol{\varepsilon}}}\{\binom{\varepsilon_1-\varepsilon_2}{\varepsilon_3-\varepsilon_2}\} = \binom{-0.3}{1.5} \\ \mathbb{V}^{f_{\boldsymbol{\varepsilon}}}\{0.5\times\varepsilon_2 + 0.7\times\varepsilon_1\} = 0.2. \end{cases} \tag{1}$$

Assume the P&L's can be approximated as an affine function of the risk drivers via Taylor pricing and that there is one cross-sectional predictive signal corresponding to the risk drivers, which is also scored.
Summarize in detail, using pseudo-code, the steps that you would apply to find the factor

premium of characteristic portfolio that stems from the signal and is in accordance with the views (1), over a one-day investment horizon.

Invariants prior *[7 points]*

Risk drivers $\{\boldsymbol{x}_t \equiv \ln \boldsymbol{v}_t^{stock}\}_{t=0}^{\bar{t}}$

Invariants $\{\boldsymbol{\epsilon}_t \equiv \boldsymbol{x}_t - \boldsymbol{x}_{t-1}\}_{t=1}^{\bar{t}}$

Invariants exp. and cov. (historical) $(\underline{\boldsymbol{\mu}}_{\boldsymbol{\varepsilon}}, \underline{\boldsymbol{\sigma}}^2_{\boldsymbol{\varepsilon}}) \leftarrow \texttt{ewma\_meancov}(\{\boldsymbol{\epsilon}_t\}_{t=1}^{\bar{t}}, w, \tau_{HL})$

Invariants posterior *[9 points]*

Initialize view specifiers

$$\begin{aligned}
&\boldsymbol{v}_{\mu} \leftarrow \mathbf{0}_{2\times\bar{n}}\\
&[\boldsymbol{v}_{\mu}]_{1,1} \leftarrow 1\\
&[\boldsymbol{v}_{\mu}]_{1,2} \leftarrow -1\\
&[\boldsymbol{v}_{\mu}]_{2,2} \leftarrow -1\\
&[\boldsymbol{v}_{\mu}]_{2,3} \leftarrow 1\\
&\boldsymbol{v}_{\sigma} \leftarrow \mathbf{0}_{1\times\bar{n}}\\
&[\boldsymbol{v}_{\sigma}]_{1} \leftarrow 0.7\\
&[\boldsymbol{v}_{\sigma}]_{2} \leftarrow 0.5
\end{aligned}$$

Initialize view quantifiers

$$\begin{aligned}
&\boldsymbol{\mu}^{view} \leftarrow \left(\begin{smallmatrix}-0.3\\1.5\end{smallmatrix}\right)\\
&\sigma^{2view} \leftarrow 0.2
\end{aligned}$$

Invariants exp. and cov. (minimum relative entropy) $(\bar{\boldsymbol{\mu}}_{\boldsymbol{\varepsilon}}, \bar{\boldsymbol{\sigma}}^2_{\boldsymbol{\varepsilon}}) \leftarrow \texttt{min\_rel\_entropy\_normal}(\underline{\boldsymbol{\mu}}_{\boldsymbol{\varepsilon}}, \underline{\boldsymbol{\sigma}}^2_{\boldsymbol{\varepsilon}}, \boldsymbol{v}_{\mu}, \boldsymbol{\mu}^{view}, \boldsymbol{v}_{\sigma}, \sigma^{2view})$

Factor premium *[9 points]*

Risk drivers volatilities

$$\begin{aligned}
&\bar{\boldsymbol{\sigma}}^2_{\boldsymbol{X}} \leftarrow \bar{\boldsymbol{\sigma}}^2_{\boldsymbol{\varepsilon}}\\
&\bar{\boldsymbol{\sigma}}^{vol}_{\boldsymbol{X}} \leftarrow \sqrt{diag(\bar{\boldsymbol{\sigma}}^2_{\boldsymbol{X}})}
\end{aligned}$$

for $t = 0, \ldots, \bar{t} - 1$

Signal beta $\boldsymbol{\beta}_{\boldsymbol{\Pi},Z;t} \leftarrow Diag(\boldsymbol{v}_t^{stock} \circ \bar{\boldsymbol{\sigma}}^{vol}_{\boldsymbol{X}})\, \boldsymbol{s}_t$

P&L's covariance $\bar{\boldsymbol{\sigma}}^2_{\boldsymbol{\Pi};t} \leftarrow Diag(\boldsymbol{v}_t^{stock}) \times \bar{\boldsymbol{\sigma}}^2_{\boldsymbol{X}} \times Diag(\boldsymbol{v}_t^{stock})$

Signal characteristic portfolio $\boldsymbol{h}_t^{ch} \leftarrow \frac{(\bar{\boldsymbol{\sigma}}^2_{\boldsymbol{\Pi};t})^{-1}\boldsymbol{\beta}_{\boldsymbol{\Pi},Z;t}}{\boldsymbol{\beta}'_{\boldsymbol{\Pi},Z;t}(\bar{\boldsymbol{\sigma}}^2_{\boldsymbol{\Pi};t})^{-1}\boldsymbol{\beta}_{\boldsymbol{\Pi},Z;t}}$

Signal-induced factor $z_{t+1}^{ch} \leftarrow \boldsymbol{h}_t^{ch\prime}(\boldsymbol{v}_{t+1}^{stock} - \boldsymbol{v}_t^{stock} - r^{rf}_{t\rightarrow t+1}\boldsymbol{v}_t^{stock})$

Premium (historical) $\lambda \leftarrow \texttt{ewma\_meancov}(\{z_t^{ch}\}_{t=1}^{\bar{t}}, w^{premium}, \tau_{HL}^{premium})$

Solution 3 Scenario-probability distribution: step 6 to 8 of the Checklist *[Tot. 30 points]*

Consider a portfolio invested in $\bar{n} \equiv 3$ financial instruments whose composition at the current time t_{now} is $\boldsymbol{h} \equiv (15,\ 20,\ 8)'$. The budget is $v_{\boldsymbol{h},t_{now}} \equiv \$1000$.
The joint scenario-probability distribution of the instruments ex-ante P&L's is described by the following $\bar{j} \equiv 4$ scenarios

$$\begin{pmatrix} \Pi_{1,t_{now}\to t_{hor}} \\ \Pi_{2,t_{now}\to t_{hor}} \\ \Pi_{3,t_{now}\to t_{hor}} \end{pmatrix} \sim \{ \underset{\boldsymbol{\pi}^{(1)}}{\underset{\uparrow}{\begin{pmatrix} \$15 \\ -\$8 \\ \$0.6 \end{pmatrix}}}, \underset{p^{(1)}}{\underset{\uparrow}{15\%}}, \quad \underset{\boldsymbol{\pi}^{(2)}}{\underset{\uparrow}{\begin{pmatrix} \$7 \\ -\$12 \\ \$0.9 \end{pmatrix}}}, \underset{p^{(2)}}{\underset{\uparrow}{45\%}}, \quad \underset{\boldsymbol{\pi}^{(3)}}{\underset{\uparrow}{\begin{pmatrix} \$8 \\ -\$6 \\ \$0.4 \end{pmatrix}}}, \underset{p^{(3)}}{\underset{\uparrow}{30\%}}, \quad \underset{\boldsymbol{\pi}^{(4)}}{\underset{\uparrow}{\begin{pmatrix} \$5 \\ -\$10 \\ \$1.2 \end{pmatrix}}}, \underset{p^{(4)}}{\underset{\uparrow}{10\%}} \}. \tag{2}$$

Suppose that the ex-ante performance is the return on value of the portfolio, i.e. $Y_{\boldsymbol{h},t_{now}\to t_{hor}} \equiv R_{\boldsymbol{h},t_{now}\to t_{hor}}$.

i) *[5 points]* Compute the scenario-probability distribution of the ex-ante performance.

The standardized holdings read $\tilde{\boldsymbol{h}} \equiv \frac{\boldsymbol{h}}{v_{\boldsymbol{h},t_{now}}} = (0.015,\ 0.02,\ 0.008)'$. The ex-ante performance scenarios can be computed as $r_{\boldsymbol{h}}^{(j)} \equiv y_{\tilde{\boldsymbol{h}}}^{(j)} = \tilde{\boldsymbol{h}}'\boldsymbol{\pi}^{(j)}$ and then the scenario-probability distribution reads

$$R_{\boldsymbol{h}} \sim \{ \underset{r_{\boldsymbol{h}}^{(1)}}{\underset{\uparrow}{6.98\%}}, \underset{p^{(1)}}{\underset{\uparrow}{15\%}}, \quad \underset{r_{\boldsymbol{h}}^{(2)}}{\underset{\uparrow}{-12.78\%}}, \underset{p^{(2)}}{\underset{\uparrow}{45\%}}, \quad \underset{r_{\boldsymbol{h}}^{(3)}}{\underset{\uparrow}{0.32\%}}, \underset{p^{(3)}}{\underset{\uparrow}{30\%}}, \quad \underset{r_{\boldsymbol{h}}^{(4)}}{\underset{\uparrow}{-11.54\%}}, \underset{p^{(4)}}{\underset{\uparrow}{10\%}} \}, \tag{3}$$

where we set, for ease of notation, $R_{\boldsymbol{h}} \equiv R_{\boldsymbol{h},t_{now}\to t_{hor}}$.

ii) *[10 points]* Compute the ex-ante performance mean-variance trade-off with variance penalty equal to $\lambda \equiv 1.2$, the certainty-equivalent determined by the exponential function $utility(y) \equiv -e^{-\lambda y}$ with $\lambda \equiv 1.2$, the information ratio and the Sortino ratio with target $r \equiv 0.2\%$.

The mean-variance trade-off with variance penalty $\lambda \equiv 1.2$ is

$$mv_{\lambda}(\boldsymbol{h}) = \mathbb{E}\{R_{\boldsymbol{h}}\} - \frac{\lambda}{2}\mathbb{V}\{R_{\boldsymbol{h}}\} = \textstyle\sum_j p^{(j)} r_{\boldsymbol{h}}^{(j)} - \frac{1.2}{2} \times (\sum_j p^{(j)} (r_{\boldsymbol{h}}^{(j)})^2 - (\sum_j p^{(j)} r_{\boldsymbol{h}}^{(j)})^2) = -0.0613. \tag{4}$$

The certainty-equivalent determined by the exponential utility function $utility(y) \equiv -e^{-\lambda y}$ with $\lambda \equiv 1.2$ is

$$cert\_eq(\boldsymbol{h}) = utility^{-1}(-\textstyle\sum_j p^{(j)} e^{-\lambda r_{\boldsymbol{h}}^{(j)}}) = -\frac{1}{\lambda}\ln(\sum_j p^{(j)} e^{-\lambda r_{\boldsymbol{h}}^{(j)}}) = -0.0612, \tag{5}$$

where $utility^{-1}(y) = -\frac{1}{\lambda}\ln(-y)$.
The information ratio is

$$ir(\boldsymbol{h}) = \frac{\mathbb{E}\{R_{\boldsymbol{h}}\}}{\mathbb{S}d\{R_{\boldsymbol{h}}\}} = \frac{\sum_j p^{(j)} r_{\boldsymbol{h}}^{(j)}}{\sqrt{\sum_j p^{(j)} (r_{\boldsymbol{h}}^{(j)})^2 - (\sum_j p^{(j)} r_{\boldsymbol{h}}^{(j)})^2}} = -0.7380. \tag{6}$$

The Sortino ratio with target $r \equiv 0.2\%$ is

$$so(\boldsymbol{h}, r) = \frac{\sum_j p^{(j)} r_{\boldsymbol{h}}^{(j)} - r}{\sqrt{\sum_j p^{(j)} |r_{\boldsymbol{h}}^{(j)} - r|^2 1_{r_{\boldsymbol{h}}^{(j)} \leq r}}} = -0.6299. \tag{7}$$

iii) *[7 points]* Consider $\bar{k} \equiv 2$ risk factors $\boldsymbol{Z}$ and suppose that their joint distribution with the ex-ante performance is

$$\begin{pmatrix} R_{\boldsymbol{h}} \\ Z_1 \\ Z_2 \end{pmatrix} \sim \{ \begin{pmatrix} 6.98\% \\ 10 \\ 20 \end{pmatrix}, \underset{p^{(1)}}{\underset{\uparrow}{15\%}}, \begin{pmatrix} -12.78\% \\ 18 \\ 25 \end{pmatrix}, \underset{p^{(2)}}{\underset{\uparrow}{45\%}}, \begin{pmatrix} 0.32\% \\ 7 \\ 18 \end{pmatrix}, \underset{p^{(3)}}{\underset{\uparrow}{30\%}}, \begin{pmatrix} -11.54\% \\ 15 \\ 28 \end{pmatrix}, \underset{p^{(4)}}{\underset{\uparrow}{10\%}} \}. \tag{8}$$

Compute the scenario-probability distribution of the residual $U_{\boldsymbol{h}}$ and factors $\boldsymbol{Z}$, where

$$U_{\boldsymbol{h}} = R_{\boldsymbol{h}} - \alpha_{\boldsymbol{h}} - \boldsymbol{\beta}_{\boldsymbol{h}} \boldsymbol{Z}, \tag{9}$$
$$\alpha_{\boldsymbol{h}} = \mathbb{E}\{R_{\boldsymbol{h}} - \boldsymbol{\beta}_{\boldsymbol{h}} \boldsymbol{Z}\}, \tag{10}$$
$$\boldsymbol{\beta}_{\boldsymbol{h}} = \mathbb{C}v\{R_{\boldsymbol{h}}, \boldsymbol{Z}\} (\mathbb{C}v\{\boldsymbol{Z}\})^{-1}. \tag{11}$$

Suppose now that $\bar{k} \equiv 3$ and that $\boldsymbol{Z} = \boldsymbol{\Pi}_{t_{now} \to t_{hor}} = (\Pi_{1,t_{now} \to t_{hor}}, \Pi_{2,t_{now} \to t_{hor}}, \Pi_{3,t_{now} \to t_{hor}})'$ with distribution as in (2). Determine the shift term $\alpha_{\boldsymbol{h}}$ (10), the exposures $\boldsymbol{\beta}_{\boldsymbol{h}}$ (11), and the residual $U_{\boldsymbol{h}}$ (9).

The exposures read

$$\boldsymbol{\beta}_{\boldsymbol{h}} = (-0.0095, \ -0.0069), \tag{12}$$

and the shift term is equal to

$$\alpha_{\boldsymbol{h}} = 0.2233. \tag{13}$$

Then, the scenario-probability distribution of the residual $U_{\boldsymbol{h}}$ and factors $\boldsymbol{Z}$ is equal to

$$\begin{pmatrix} U_{\boldsymbol{h}} \\ Z_1 \\ Z_2 \end{pmatrix} \sim \{ \begin{pmatrix} 0.08 \\ 10 \\ 20 \end{pmatrix}, \underset{p^{(1)}}{\underset{\uparrow}{15\%}}, \begin{pmatrix} -0.0069 \\ 18 \\ 25 \end{pmatrix}, \underset{p^{(2)}}{\underset{\uparrow}{45\%}}, \begin{pmatrix} -0.0289 \\ 7 \\ 18 \end{pmatrix}, \underset{p^{(3)}}{\underset{\uparrow}{30\%}}, \begin{pmatrix} -0.0022 \\ 15 \\ 28 \end{pmatrix}, \underset{p^{(4)}}{\underset{\uparrow}{10\%}} \}, \tag{14}$$

where $u_{\boldsymbol{h}}^{(j)} \equiv r^{(j)} - \alpha_{\boldsymbol{h}} - \boldsymbol{\beta}_{\boldsymbol{h}} \boldsymbol{z}^{(j)}$.

When $\boldsymbol{Z} = \boldsymbol{\Pi}_{t_{now} \to t_{hor}}$, the portfolio exposures are the standardized holdings, i.e. $\boldsymbol{\beta}_{\boldsymbol{h}} \equiv \tilde{\boldsymbol{h}}'$, and the attribution model is exact, i.e. $\alpha_{\boldsymbol{h}} = 0$ and $U_{\boldsymbol{h}} = 0$.

iv) *[8 points]* Suppose that the index of satisfaction is $-\mathbb{V}\{R_{\boldsymbol{h}}\}$. Then, compute the "first in" proportional attribution terms $[-\mathbb{V}\{R_{\boldsymbol{h}}\}]_k^{isol}$ for $k = 0, 1, 2$ associated to each risk factor Z_k, where for $k = 0$ the risk factor is defined as $Z_0 = \frac{1}{\mathbb{S}d\{U_{\boldsymbol{h}}\}}(\alpha_{\boldsymbol{h}} + U_{\boldsymbol{h}})$ and the corresponding exposure is equal to $\beta_0 = \mathbb{S}d\{U_{\boldsymbol{h}}\}$.

Does the fact that the individual contributions sum up to $-\mathbb{V}\{R_{\boldsymbol{h}}\}$ rely on the fact that $R_{\boldsymbol{h}}$ is a linear combination of the factors $\boldsymbol{Z}$?

The scenario-probability distribution of $\boldsymbol{Z} \equiv (Z_0, Z_1, Z_2)'$, with $Z_0 = \frac{1}{\mathbb{S}d\{U_{\boldsymbol{h}}\}}(\alpha_{\boldsymbol{h}} + U_{\boldsymbol{h}})$, is

$$\begin{pmatrix} Z_0 \\ Z_1 \\ Z_2 \end{pmatrix} \sim \{ \begin{pmatrix} 8.6367 \\ 10 \\ 20 \end{pmatrix}, \underset{p^{(1)}}{\underset{\uparrow}{15\%}}, \quad \begin{pmatrix} 6.1609 \\ 18 \\ 25 \end{pmatrix}, \underset{p^{(2)}}{\underset{\uparrow}{45\%}}, \quad \begin{pmatrix} 5.5349 \\ 7 \\ 18 \end{pmatrix}, \underset{p^{(3)}}{\underset{\uparrow}{30\%}}, \quad \begin{pmatrix} 6.2946 \\ 15 \\ 28 \end{pmatrix}, \underset{p^{(4)}}{\underset{\uparrow}{10\%}} \}, \tag{15}$$

where $z_0^{(j)} \equiv \frac{\alpha_{\boldsymbol{h}} + u_{\boldsymbol{h}}^{(j)}}{\mathbb{S}d\{U_{\boldsymbol{h}}\}}$. We now set $\boldsymbol{\beta} \equiv (\beta_0, \beta_1, \beta_2) = (0.0351, \ -0.0095, \ -0.0069)$ where we drop the subscript $\boldsymbol{h}$ for ease of notation.
The terms $\mathbb{V}\{\beta_k Z_k\}$ follow from affine equivariance, i.e. $\mathbb{V}\{\beta_k Z_k\} = \beta_k^2 \mathbb{V}\{Z_k\}$, and read

$$\mathbb{V}\{\beta_0 Z_0\} = 0.0012, \ \mathbb{V}\{\beta_1 Z_1\} = 0.0021, \ \mathbb{V}\{\beta_2 Z_2\} = 6.1694 \times 10^{-4}. \tag{16}$$

The normalization coefficient reads

$$\gamma^{isol} \equiv \frac{-\mathbb{V}\{R_{\boldsymbol{h}}\}}{-\mathbb{V}\{\beta_0 Z_0\} - \mathbb{V}\{\beta_1 Z_1\} - \mathbb{V}\{\beta_2 Z_2\}} = 1.526, \tag{17}$$

and, then, the "first in" proportional attribution terms are equal to

$$[-\mathbb{V}\{R_{\boldsymbol{h}}\}]_0^{isol} = -\gamma^{isol} \times \mathbb{V}\{\beta_0 Z_0\} = -0.0019, \tag{18}$$

$$[-\mathbb{V}\{R_{\boldsymbol{h}}\}]_1^{isol} = -\gamma^{isol} \times \mathbb{V}\{\beta_1 Z_1\} = -0.0033, \tag{19}$$

$$[-\mathbb{V}\{R_{\boldsymbol{h}}\}]_2^{isol} = -\gamma^{isol} \times \mathbb{V}\{\beta_2 Z_2\} = -9.4144 \times 10^{-4}. \tag{20}$$

The fact that the individual contributions sum up to $-\mathbb{V}\{R_{\boldsymbol{h}}\}$, i.e.

$$\textstyle\sum_k [-\mathbb{V}\{R_{\boldsymbol{h}}\}]_k^{isol} = -0.0061 = -\mathbb{V}\{R_{\boldsymbol{h}}\}, \tag{21}$$

does not rely on the linearity of the ex-ante performance with respect to the risk factors $\boldsymbol{Z}$, because the methodology is fully general.

Solution 4 Expectation and variance of the market impact P&L *[Tot. 15 points]*

Consider the Obizhaeva-Wang market impact model for the volume time $q \in [0, q_{end}]$

$$\hat{P}_q = p_0 + \sigma B_q + \gamma \textstyle\int_0^{q_{end}} e^{-\rho(q-s)} \dot{h}_s ds, \tag{22}$$

where B is a Brownian motion and σ, γ and ρ are positive constants.
Consider the round trip trading strategy h_q, for $q \in [0, q_{end}]$,

$$h_q = \begin{cases} \nu q & \text{for } 0 \leq q \leq \frac{q_{end}}{2} \\ \nu(q_{end} - q) & \text{for } \frac{q_{end}}{2} \leq q \leq q_{end} \end{cases}, \tag{23}$$

where ν is a positive constant.

i) *[5 points]* Determine the variance of the market impact P&L $\hat{\Pi}_{0 \to q_{end}}$ generated by the round trip strategy.

For the given round trip strategy we have

$$\mathbb{V}\{\hat{\Pi}_{0\to q_{end}}\} = \sigma^2 \int_{q_{now}}^{q_{end}} h_q^2 dq = \sigma^2\nu^2 \int_0^{\frac{q_{end}}{2}} q^2 dq + \sigma^2\nu^2 \int_{\frac{q_{end}}{2}}^{q_{end}} (q_{end} - q)^2 dq$$
$$= \sigma^2\nu^2 \frac{q_{end}^3}{12}. \tag{24}$$

ii) *[10 points]* Does the variance $\mathbb{V}\{\hat{\Pi}_{0\to q_{end}}\}$ change if we consider the Almgren-Chriss market impact model?

The variance $\mathbb{V}\{\hat{\Pi}_{0\to q_{end}}\}$ does not change because it is model independent. Indeed, it depends only on how the trading strategy h_q is defined.