ARPM Certificate

Level 2

Example

This is an example of examination sheet for the Certificate in Advanced Risk and Portfolio Management (ARPM Certificate) - Level 2.

The exercises below test the following modules from the body of knowledge:
- Quantitative Risk Management
- Quantitative Portfolio Management.

The exam is graded according to the breakdown policy specified on the examination sheet. The grade for the exam is a number L_2 in the interval $[0, 1]$, computed via an increasing function $L_2(p)$, where p is the total number of points that you collect by solving the exercises below. The minimum score to pass is 0.65.

Exercise 1 True or false? Justify your answer *[Tot. 30 points]*

i) *[3 points]* A random variable X strongly dominates another random variable Y if and only if X dominates Y weakly as well.

ii) *[3 points]* If an index of satisfaction (or a risk measure) is consistent with weak dominance, then it is also consistent with strong dominance.

iii) *[3 points]* The ex-ante performance $Y_{\boldsymbol{h}}$, say the P&L or the linear return or the excess linear return of a portfolio, is always affine in terms of the holdings $\boldsymbol{h}$.

iv) *[3 points]* Quantile-based indices of satisfaction or risk measures for a buy-and-hold portfolio, such as the VaR and the CVaR, are all positive homogeneous of degree 1 in the standardized holdings.

v) *[3 points]* The mean-variance trade-off is an index of satisfaction which is popular and commonly used in practice because it satisfies many preferrable properties such as monotonicity, positive homogeneity and co-monotonic additivity.

vi) *[3 points]* The buy-and-hold strategy stemming from the expected utility maximization has a concave payoff profile.

vii) *[3 points]* In the Black-Litterman approach, the views are statements on the covariances of the returns on some portfolios.

viii) *[3 points]* The signal characteristic portfolio is a pseudo-inverse of the signal beta.

ix) *[3 points]* In the backward step-wise selection routine we build the optimal combination by removing step by step the elements which give rise to the lowest value of the objective function.

x) *[3 points]* When scheduling to optimally liquidate a parent order, it is most natural to plan the schedule in tick time since historical trades and quotes are recorded in tick time.

Exercise 2 Construction: embedding views and advanced portfolio construction *[Tot. 25 points]*

Consider as daily risk drivers the log-values of the 500 stocks in the S&P 500.
Assume that the risk drivers follow a multivariate random walk process with multivariate normal shocks over a daily time step.
Assume that we have the following partial views on the invariants distribution

$$f_{\boldsymbol{\varepsilon}} \in \mathcal{V}_{\boldsymbol{\varepsilon}}: \qquad \begin{cases} \mathbb{E}^{f_{\boldsymbol{\varepsilon}}}\{\left(\begin{smallmatrix}\varepsilon_1-\varepsilon_2\\ \varepsilon_3-\varepsilon_2\end{smallmatrix}\right)\} = \left(\begin{smallmatrix}-0.3\\ 1.5\end{smallmatrix}\right) \\ \mathbb{V}^{f_{\boldsymbol{\varepsilon}}}\{0.5\times\varepsilon_2 + 0.7\times\varepsilon_1\} = 0.2. \end{cases} \tag{1}$$

Assume the P&L's can be approximated as an affine function of the risk drivers via Taylor pricing and that there is one cross-sectional predictive signal corresponding to the risk drivers, which is also scored.
Summarize in detail, using pseudo-code, the steps that you would apply to find the factor premium of characteristic portfolio that stems from the signal and is in accordance with the views (1), over a one-day investment horizon.

Exercise 3 Scenario-probability distribution: step 6 to 8 of the Checklist *[Tot. 30 points]*

Consider a portfolio invested in $\bar{n} \equiv 3$ financial instruments whose composition at the current time t_{now} is $\boldsymbol{h} \equiv (15,\ 20,\ 8)'$. The budget is $v_{\boldsymbol{h},t_{now}} \equiv \$1000$.
The joint scenario-probability distribution of the instruments ex-ante P&L's is described by the following $\bar{j} \equiv 4$ scenarios

$$\begin{pmatrix}\Pi_{1,t_{now}\to t_{hor}}\\ \Pi_{2,t_{now}\to t_{hor}}\\ \Pi_{3,t_{now}\to t_{hor}}\end{pmatrix} \sim \{\underbrace{\begin{pmatrix}\$15\\ -\$8\\ \$0.6\end{pmatrix}}_{\boldsymbol{\pi}^{(1)}}, \underbrace{15\%}_{p^{(1)}},\quad \underbrace{\begin{pmatrix}\$7\\ -\$12\\ \$0.9\end{pmatrix}}_{\boldsymbol{\pi}^{(2)}}, \underbrace{45\%}_{p^{(2)}},\quad \underbrace{\begin{pmatrix}\$8\\ -\$6\\ \$0.4\end{pmatrix}}_{\boldsymbol{\pi}^{(3)}}, \underbrace{30\%}_{p^{(3)}},\quad \underbrace{\begin{pmatrix}\$5\\ -\$10\\ \$1.2\end{pmatrix}}_{\boldsymbol{\pi}^{(4)}}, \underbrace{10\%}_{p^{(4)}}\}. \tag{2}$$

Suppose that the ex-ante performance is the return on value of the portfolio, i.e. $Y_{\boldsymbol{h},t_{now}\to t_{hor}} \equiv R_{\boldsymbol{h},t_{now}\to t_{hor}}$.

i) *[5 points]* Compute the scenario-probability distribution of the ex-ante performance.

ii) *[10 points]* Compute the ex-ante performance mean-variance trade-off with variance penalty equal to $\lambda \equiv 1.2$, the certainty-equivalent determined by the exponential function $utility(y) \equiv -e^{-\lambda y}$ with $\lambda \equiv 1.2$, the information ratio and the Sortino ratio with target $r \equiv 0.2\%$.

iii) *[7 points]* Consider $\bar{k} \equiv 2$ risk factors $\boldsymbol{Z}$ and suppose that their joint distribution with the ex-ante performance is

$$\begin{pmatrix} R_{\boldsymbol{h}} \\ Z_1 \\ Z_2 \end{pmatrix} \sim \{ \begin{pmatrix} 6.98\% \\ 10 \\ 20 \end{pmatrix}, \underset{p^{(1)}}{\underset{\uparrow}{15\%}}, \quad \begin{pmatrix} -12.78\% \\ 18 \\ 25 \end{pmatrix}, \underset{p^{(2)}}{\underset{\uparrow}{45\%}}, \quad \begin{pmatrix} 0.32\% \\ 7 \\ 18 \end{pmatrix}, \underset{p^{(3)}}{\underset{\uparrow}{30\%}}, \quad \begin{pmatrix} -11.54\% \\ 15 \\ 28 \end{pmatrix}, \underset{p^{(4)}}{\underset{\uparrow}{10\%}} \}. \tag{3}$$

Compute the scenario-probability distribution of the residual $U_{\boldsymbol{h}}$ and factors $\boldsymbol{Z}$, where

$$U_{\boldsymbol{h}} = R_{\boldsymbol{h}} - \alpha_{\boldsymbol{h}} - \boldsymbol{\beta}_{\boldsymbol{h}} \boldsymbol{Z}, \tag{4}$$

$$\alpha_{\boldsymbol{h}} = \mathbb{E}\{R_{\boldsymbol{h}} - \boldsymbol{\beta}_{\boldsymbol{h}} \boldsymbol{Z}\}, \tag{5}$$

$$\boldsymbol{\beta}_{\boldsymbol{h}} = \mathbb{C}v\,\{R_{\boldsymbol{h}}, \boldsymbol{Z}\}\,(\mathbb{C}v\,\{\boldsymbol{Z}\})^{-1}. \tag{6}$$

Suppose now that $\bar{k} \equiv 3$ and that $\boldsymbol{Z} = \boldsymbol{\Pi}_{t_{now} \to t_{hor}} = (\Pi_{1,t_{now} \to t_{hor}}, \Pi_{2,t_{now} \to t_{hor}}, \Pi_{3,t_{now} \to t_{hor}})'$ with distribution as in (2). Determine the shift term $\alpha_{\boldsymbol{h}}$ (4), the exposures $\boldsymbol{\beta}_{\boldsymbol{h}}$ (5), and the residual $U_{\boldsymbol{h}}$ (6).

iv) *[8 points]* Suppose that the index of satisfaction is $-\mathbb{V}\{R_{\boldsymbol{h}}\}$. Then, compute the "first in" proportional attribution terms $[-\mathbb{V}\{R_{\boldsymbol{h}}\}]_k^{isol}$ for $k = 0, 1, 2$ associated to each risk factor Z_k, where for $k = 0$ the risk factor is defined as $Z_0 = \frac{1}{\mathbb{S}d\{U_{\boldsymbol{h}}\}}(\alpha_{\boldsymbol{h}} + U_{\boldsymbol{h}})$ and the corresponding exposure is equal to $\beta_0 = \mathbb{S}d\{U_{\boldsymbol{h}}\}$.
Does the fact that the individual contributions sum up to $-\mathbb{V}\{R_{\boldsymbol{h}}\}$ rely on the fact that $R_{\boldsymbol{h}}$ is a linear combination of the factors $\boldsymbol{Z}$?

Exercise 4 Expectation and variance of the market impact P&L *[Tot. 15 points]*

Consider the Obizhaeva-Wang market impact model for the volume time $q \in [0, q_{end}]$

$$\hat{P}_q = p_0 + \sigma B_q + \gamma \int_0^{q_{end}} e^{-\rho(q-s)} \dot{h}_s ds, \tag{7}$$

where B is a Brownian motion and σ, γ and ρ are positive constants.
Consider the round trip trading strategy h_q, for $q \in [0, q_{end}]$,

$$h_q = \begin{cases} \nu q & \text{for } 0 \leq q \leq \frac{q_{end}}{2} \\ \nu(q_{end} - q) & \text{for } \frac{q_{end}}{2} \leq q \leq q_{end} \end{cases}, \tag{8}$$

where ν is a positive constant.
i) *[5 points]* Determine the variance of the market impact P&L $\hat{\Pi}_{0 \to q_{end}}$ generated by the round trip strategy.

ii) *[10 points]* Does the variance $\mathbb{V}\{\hat{\Pi}_{0 \to q_{end}}\}$ change if we consider the Almgren-Chriss market impact model?