ARPM Certificate

Level 1

Example

This is an example of examination sheet for the Certificate in Advanced Risk and Portfolio Management (ARPM Certificate) - Level 1.

The exercises below test the following modules from the body of knowledge:
- Mathematics Foundations
- Data Science Foundations
- Data Science for Finance
- Advanced Data Science.

The exam is graded according to the breakdown policy specified on the examination sheet. The grade for the exam is a number L_1 in the interval $[0, 1]$, computed via an increasing function $L_1(p)$, where p is the total number of points that you collect by solving the exercises below. The minimum score to pass is 0.65.

Exercise 1 True or false? Justify your answers

Note: 1 point is assigned to correct True/False; 3 points are assigned to a correct motivation.

1. *[4 points]* The lognormal distribution is stable.

2. *[4 points]* The sample mean - sample covariance ellipsoid is the minimum volume ellipsoid that includes all the data points.

3. *[4 points]* In a GARCH(1,1) process, the next-step volatility conditional on the current information is deterministic.

4. *[4 points]* The Ornstein-Uhlenbeck process is the continuous time generalization of the AR(1) process with normal shocks.

5. *[4 points]* The r-squared $\mathcal{R}^2\{Y\|X\} \equiv 1 - \frac{\mathbb{E}\{(X-Y)^2\}}{\mathbb{V}\{X\}}$ is a divergence.

6. *[4 points]* If we apply principal component analysis to a highly correlated market, the first three principal components can be interpreted as level, slope and curvature, due to the shape of the eigenvectors.

7. *[4 points]* Binary classification problems are instances of supervised learning where the input is a Bernoulli random variable.

8. *[4 points]* The ROC curve lies on the unit square and always starts from the point $(1, 1)$ and ends in the point $(0, 0)$.

9. *[4 points]* Feature engineering is the process of transforming input variables to achieve a model with better out of sample performance (variance reduction).

10. *[4 points]* Lasso is a regularization technique that can be used to perform factor selection in a regression linear factor model.

Exercise 2 Generating scenarios from a copula-marginal distribution

[20 points] Using pseudo-code, write the steps to generate scenarios representing a random variable $\boldsymbol{X} \equiv (X_1, X_2)'$ with Cauchy copula $\textit{Cauchy}(\boldsymbol{\varrho}^2)$ and the following marginal distributions:
- $X_1 \sim \textit{Bernoulli}(0.3)$
- $X_2 \sim N(1, 1)$.

You can assume that the following tools are available:
- a univariate uniform random number generator;
- a multivariate standard normal random number generator;
- cumulative distribution functions and quantile functions of the relevant univariate distributions;
- function that computes the Riccati root of a symmetric and positive semidefinite matrix.

Exercise 3 Regression linear factor model

Consider a regression linear factor model for an $(\bar{n} = 2)$-dimensional target variable $\boldsymbol{X}$ with $\bar{k} = 1$ factor Z

$$\boldsymbol{X} = \boldsymbol{\alpha} + \boldsymbol{\beta} Z + \mathring{\boldsymbol{\varepsilon}}. \tag{1}$$

Assume that the target variables and factor have the following joint Student t distribution

$$\begin{pmatrix} X_1 \\ X_2 \\ Z \end{pmatrix} \sim t(\underbrace{\begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}}_{\boldsymbol{\mu}}, \underbrace{\begin{pmatrix} 10 & 0 & 6 \\ 0 & 6.75 & 6.5 \\ 6 & 6.5 & 20 \end{pmatrix}}_{\boldsymbol{\sigma}^2}, \underbrace{\ 4\ }_{\nu}). \tag{2}$$

1. *[10 points]* Compute the loadings $\boldsymbol{\beta} \equiv (\beta_1, \beta_2)$ that maximize the multivariate r-squared and the shift parameters $\boldsymbol{\alpha} \equiv (\alpha_1, \alpha_2)'$ such that the residuals have zero expectation.

2. *[6 points]* Show that the linear factor model (1) is *not* idiosyncratic.

3. *[6 points]* How would the results at the previous points change if the distribution of the target variables and factor were normal, with same location $\boldsymbol{\mu}$, and dispersion $2\boldsymbol{\sigma}^2$?

Exercise 4 Homogeneous Markov chain for rating transitions

Consider a homogeneous Markov chain X_t that models the rating transitions of an obligor. The Markov chain has 8 states: {"AAA", "AA", "A", "BBB", "BB", "B", "CCC", "D"}, and the following one-year transition matrix

$$\boldsymbol{p} = \begin{pmatrix} 92.58\% & 6.36\% & 0.78\% & 0.23\% & 0.05\% & 0.00\% & 0.00\% & 0.00\% \\ 2.95\% & 92.15\% & 3.99\% & 0.73\% & 0.17\% & 0.01\% & 0.00\% & 0.00\% \\ 0.24\% & 4.60\% & 89.93\% & 4.60\% & 0.55\% & 0.06\% & 0.01\% & 0.01\% \\ 0.06\% & 0.50\% & 8.22\% & 87.10\% & 3.30\% & 0.54\% & 0.14\% & 0.14\% \\ 0.07\% & 0.26\% & 1.39\% & 10.72\% & 84.46\% & 2.62\% & 0.25\% & 0.23\% \\ 0.01\% & 0.03\% & 0.34\% & 2.11\% & 11.74\% & 81.97\% & 1.90\% & 1.90\% \\ 0.00\% & 0.01\% & 0.10\% & 1.61\% & 3.67\% & 7.10\% & 82.43\% & 5.08\% \\ 0.00\% & 0.00\% & 0.00\% & 0.00\% & 0.00\% & 0.00\% & 0.00\% & 100.00\% \end{pmatrix}. \quad (3)$$

1. *[6 points]* What properties does the transition probability matrix $\boldsymbol{p}$ satisfy?
2. *[6 points]* Assume that the current rating is "BB". Compute the probability that the obligor, over the next year:
   a) stays in the same rating class;
   b) defaults;
   c) upgrades to a better rating.
3. *[6 points]* Assume that the current rating is "BB". Suppose that we know the realizations of the invariant of the Markov chain for the next year is

   $$\epsilon_{t+1} = 0.08.$$

   What is the rating of the obligor after 1 year?