ARPM Certificate

Level 2

Example

Solution 1 True or false? Justify your answers
Note: 1 point is assigned to correct True/False; 4 points are assigned to a correct motivation.

1. *[5 points]* According to the law of one price, if two European call options on the same underlying have the same strike, they should have the same value.

   False. It would be true if the two options also had the same expiry date.

2. *[5 points]* Given a large sample of weekly observations of the price of a stock, the sample standard deviation provides a good estimate for the volatility of the stock value over a one-week horizon.

   False. The price of a stock is not i.i.d. across time. Therefore, the sample standard deviation of past prices is not informative about the future volatility.

3. *[5 points]* The duration of a bond is the average time it takes to receive all the cash flows of a bond, weighted by the present value of each of the cash flows.

   True. Given a bond that pays coupons c_k at times t_k, the duration at the current time can be written as

   $$dur = \textstyle\sum_{r_k \geq s(t_{now})} p_k \tau_k$$

   where

   $$\tau_k \equiv (t_k - t_{now})$$
   $$p_k \equiv \frac{c_k e^{-\tau_k y_{t_{now}}(\tau_k)}}{\sum_{r_j \geq s(t)} c_j e^{-\tau_j y_{t_{now}}(\tau_j)}}.$$

4. *[5 points]* Consider two normal random variables $X_1 \sim N(1,1)$ and $X_2 \sim N(0,5)$. The random variable X_1 strongly dominates X_2.

   False. Even if the expectation of X_2 is smaller, there are scenarios where $X_2(\omega) > X_1(\omega)$, so neither of the two variables strongly dominates each other.

5. *[5 points]* If we consider the P&L as measure of performance, the information ratio is a positive homogeneous (of order one) measure of satisfaction.

   False. The information ratio is scale invariant: if we multiply the holdings by a positive constant we obtain

   $$\mathbb{IR}\{\Pi_{\gamma \boldsymbol{h}}\} \equiv \frac{\mathbb{E}\{\Pi_{\gamma \boldsymbol{h}}\}}{\mathbb{S}d\{\Pi_{\gamma \boldsymbol{h}}\}} = \frac{\mathbb{E}\{\gamma \boldsymbol{h}' \boldsymbol{\Pi}\}}{\mathbb{S}d\{\gamma \boldsymbol{h}' \boldsymbol{\Pi}\}} = \mathbb{IR}\{\Pi_{\boldsymbol{h}}\}. \tag{1}$$

6. *[5 points]* The function $f(x) = 3(x-1)^2$ can be used as a spectrum to define a coherent spectral measure of satisfaction/risk.

   True. The function is positive and integrates to 1 in the interval $[0,1]$, so it can be used as a spectrum. In addition, it is decreasing in the interval $[0,1]$, so it defines a coherent measure.

7. *[5 points]* The ex-ante return of a portfolio can be attributed linearly to a set of risk factors by means of the Euler decomposition.

   False. The Euler decomposition can be used for ex-ante *risk* (not performance) attribution, if the risk measure considered is positive homogeneous.

8. *[5 points]* The allocation obtained with the resampling approach may not satisfy the investment constraints.

   True. Consider the constraint of not investing in more than $\bar{k}$ of the $\bar{n}$ instruments in the market: each allocation satisfies this constraint, but the allocation computed as their average does not.

9. *[5 points]* In the context of cross-sectional strategies, signal characteristics are the regression loadings of the risk drivers $\boldsymbol{X}_{t+1}$ on the signals $\boldsymbol{S}_t$.

   False. The signal characteristics are the regression loadings of the P&L's $\boldsymbol{\Pi}_{t+1}$ on the signal-induced factor Z^{signal}_{t+1}.

10. *[5 points]* Constant proportion portfolio insurance is a time series strategy that guarantees the value stays above a minimum target.

    True from a theoretical perspective (unless we consider a model with jumps for the value). In practice the value is not guaranteed because rebalancing occurs at discrete time. Also, transaction costs to implement the strategy may have a huge impact on the overall profitability.

Solution 2 The Checklist: historical projection and pricing at the horizon

1. *[7 points]* Assume that the log-value of an equity share $X_t \equiv \ln V^{stock}_t$ follows a random walk $X_{t+1} = X_t + \varepsilon_{t+1}$ and that the shock ε_t is modeled via its historical distribution with flexible probabilities $\{\epsilon_t, p_t\}_{t=1}^{\bar{t}}$.
   Write the scenario-probability distribution of the P&L $\Pi^{stock}_{t_{now}\to t_{hor}}$ at the horizon $t_{hor} = t_{now} + 1$ day, as function of the realizations $\{\epsilon_t\}_{t=1}^{\bar{t}}$.

   The scenario-probability distribution of the P&L reads

   $$\Pi^{stock}_{t_{now}\to t_{now}+1} \sim \{\pi^{(j),stock} \equiv v^{stock}_{t_{now}} \times (e^{\epsilon_j} - 1), p^{(j)} \equiv p_j\}_{j=1}^{\bar{t}}. \tag{2}$$

2. *[7 points]* Assume now that $X_t \equiv \ln V^{stock}_t$ follows a GARCH$(1,1)$ model, where the residual ε_t is modeled via its historical distribution with flexible probabilities $\{\epsilon_t, p_t\}_{t=1}^{\bar{t}}$.
   Write the scenario-probability distribution of the P&L $\Pi^{stock}_{t_{now}\to t_{hor}}$ at the horizon $t_{hor} =$

$t_{now}+1$ day, as function of the realizations $\{\epsilon_t\}_{t=1}^{\bar{t}}$.

The scenario-probability distribution of the P&L reads

$$\Pi^{stock}_{t_{now}\rightarrow t_{now}+1} \sim \{\pi^{(j),stock} \equiv v^{stock}_{t_{now}} \times (\exp\{\mu + \sigma_{t_{now}+1}\epsilon_j\} - 1), p^{(j)} \equiv p_j\}_{j=1}^{\bar{t}}, \tag{3}$$

where $\sigma_{t_{now}+1} \equiv \sqrt{c + b\sigma^2_{t_{now}} + a(x_{t_{now}} - x_{t_{now}-1} - \mu)^2}$.

Solution 3 The Checklist: risk and portfolio management steps

[20 points] Assume that, by applying the steps of the Checklist, you obtained the scenario-probability distribution of the ex-ante P&Ls of 500 stocks over the investment horizon

$$\mathbf{\Pi} \sim \{\boldsymbol{\pi}^{(j)}, p^{(j)}\}_{j=1}^{\bar{j}}. \tag{4}$$

Using pseudo-code, write the steps that you would use to set up and solve a mean-variance optimization problem, where:
- the investment constraints are the full investment of the budget and no short selling;
- the risk/satisfaction is measured by the expected shortfall of the portfolio *return* distribution at confidence level 90% (you can assume to have a function that computes spectral measures of satisfaction given the spectrum and the distribution of the ex-ante performance).

1. Compute the expectation and covariance of the P&L distribution

$\mathbb{E}\{\mathbf{\Pi}\}, \mathbb{C}v\{\mathbf{\Pi}\} \leftarrow$`meancov_sp(`$\{\boldsymbol{\pi}^{(j)}, p^{(j)}\}_{j=1}^{\bar{j}}$`)`

2. Compute the expectation and covariance of the distribution of the return $\boldsymbol{R} \equiv Diag(1./\boldsymbol{v})\,\mathbf{\Pi}$ by affine equivariance

$\mathbb{E}\{\boldsymbol{R}\} \leftarrow Diag(1./\boldsymbol{v})\ \mathbb{E}\{\mathbf{\Pi}\}$
$\mathbb{C}v\{\boldsymbol{R}\} \leftarrow Diag(1./\boldsymbol{v})\mathbb{C}v\{\mathbf{\Pi}\}Diag(1./\boldsymbol{v})'$

3. Solve the first step of the mean-variance optimization problem over a suitable grid of values for the trade-off parameter λ, i.e. compute the efficient frontier

$\{\lambda_l\}_{l=1}^{\bar{l}} \leftarrow$`(from` $\lambda^{\inf}$`, to` $\lambda^{\sup}$`, step=` λ^{step}`)`
$\mathcal{A} \leftarrow \{\boldsymbol{w} : \boldsymbol{w}'\mathbf{1} = 1\}$
$\mathcal{C} \leftarrow \{\boldsymbol{w} : -\boldsymbol{w} \leq \mathbf{0}\}$
for $l = 1, ..., \bar{l}$
$\quad \boldsymbol{w}_{\lambda_l} \leftarrow \underset{\boldsymbol{w}\in\mathcal{A}\cap\mathcal{C}}{\text{argmax}}(\boldsymbol{w}'\mathbb{E}\{\boldsymbol{R}\} - \lambda_l \times \boldsymbol{w}'\mathbb{C}v\{\boldsymbol{R}\}\boldsymbol{w})$
end

4. Solve the second step of the mean-variance optimization problem, i.e. minimize the expected shortfall over the efficient frontier

$c \leftarrow 0.9$
for $l = 1, ..., \bar{l}$
    $r^{(j)}_{\boldsymbol{w}_{\lambda_l}} \leftarrow \boldsymbol{w}'_{\lambda_l} \boldsymbol{r}^{(j)}$
    $spectr(x) \leftarrow \frac{1}{1-c} \cdot 1_{x \in [0, 1-c]}$
    $-\mathbb{ES}_c\{R_{\boldsymbol{w}_{\lambda_l}}\} \leftarrow$ `spectral_satis(`$spectr(\cdot)$`, `$\{r^{(j)}_{\boldsymbol{w}_{\lambda_l}}, p^{(j)}\}_{j=1}^{\bar{j}}$`)`
end
$\lambda^* \leftarrow \underset{\lambda \in \{\lambda_l\}_{l=1}^{\bar{l}}}{\text{argmax}} -\mathbb{ES}_c\{R_{\boldsymbol{w}_{\lambda_l}}\}$
$\boldsymbol{w}^{qsi*} \leftarrow \boldsymbol{w}_{\lambda^*}$

Solution 4 Market buy/sell liquidity curve.
[16 points] Define the market buy and market sell liquidity curves and briefly summarize their interpretation.

The market buy liquidity curve takes as input a positive number of shares $\Delta h > 0$ and outputs the price

$$P^{mb}_t(\Delta h) \equiv \min\{p \text{ such that } \Delta H^{mb}_t(p) \geq \Delta h\},$$

where $\Delta H^{mb}_t(p)$ is the number of shares sitting between the best ask P^{ask}_t and the higher price p. Intuitively, the market buy liquidity curve tells us what would be the new ask price if a market order to buy Δh shares was placed.

Similarly, the market sell liquidity curve takes as input a negative number of shares $\Delta h < 0$ and outputs the price

$$P^{ms}_t(\Delta h) \equiv \max\{p \text{ such that } \Delta H^{ms}_t(p) \leq \Delta h\},$$

where $\Delta H^{ms}_t(p)$ is the number of shares sitting between the best bid P^{bid}_t and the lower price p. Intuitively, the market sell liquidity curve tells us what would be the new bid price if a market order to sell $|\Delta h|$ shares was placed.